

# Free scalar dark matter candidates in $R^2$ -inflation: the light, the heavy and the superheavy

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## Abstract

Gravity takes care of both inflation and subsequent reheating in Starobinsky's  $R^2$ -model. The latter is due to inflaton gravitation decays dominated by scalar particle production. It is tempting to suggest that dark matter particles are also produced in this process. Since free scalars being too hot cannot serve as viable dark matter [3], we further study the issue considering two options: scalars with non-minimal coupling to gravity and superheavy scalars generated at inflationary stage. We found that the first option allows for viable warm or cold dark matter if scalar mass exceeds 1.1 MeV. The second option implies supercold dark matter with particle mass  $10^{16}$  GeV, which production is saturated at the end of inflation when inflaton-dependent scalar mass rapidly changes and violates adiabaticity. Similar result holds for superheavy fermion dark matter.

Cosmology of the homogeneous and isotropic Universe in Starobinsky's  $R^2$ -model [1] has an inflationary stage. This stage is realized as a large-field slow-roll inflation driven by scalaron field, that is a scalar degree of freedom coming from gravity sector of the theory. Subsequent decay of the scalarons into ordinary matter dominated by scalar particles production reheats the Universe. It is natural to speculate that dark matter particles could be produced in the same way.

Indeed, it is gravity—the mostly universal force—that provides interaction for the scalaron with all (matter) fields. The strength of coupling to a given field is determined by the strength

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of conformal symmetry violation. This leads to the fact that mass terms, being conformally noninvariant, become modulated by the scalaron field. Moreover, scalars have extra coupling due to their conformally-noninvariant kinetic term. Consequently, the scalaron decays are dominated by scalar particles production [1, 2], so that free scalars produced in an appropriate amount would be too hot and could not serve as the dark matter [3]. At the same time, free heavy fermions can do the job. However, scalar dark matter implies only one new degree of freedom to be added to the SM, and therefore is the most economic solution.

There are two obvious ways to get rid of the problem with scalar dark matter and avoid over-energetic scalar relics. First, one can partially restore the conformal symmetry by adding nonminimal coupling of scalar to gravity. By changing the value of corresponding coupling constant  $\xi$  one can control the strength of conformal symmetry violation and govern the scalaron decay rate into dark matter particles. Then heavier and hence colder scalar relics become viable dark matter. The second option is to consider superheavy scalar particles which kinematically can not be produced from scalaron decays at postinflationary stage. However, some amount of them can be generated nonperturbatively in the time-dependent scalaron background.

In this work we consider both options. We obtain the mass of the scalar dark matter candidate as a function of nonminimal coupling to gravity  $\xi$ . Depending on the value of  $\xi$  dark matter produced at postinflationary epoch can be light or heavy, the lightest viable free scalars are of 1.1 MeV and form warm dark matter. Such particular candidate is interesting in the context of emerging problems of the standard CDM cosmology on small scales, such as missing satellites, galactic matter density profiles and angular momentum of spiral galaxies, for review see e.g. [4].

Considering the second option we find that the production of superheavy particles is saturated at the end of inflation, when the scalar mass modulated by the scalaron field changes rapidly. A nontrivial result is that the mass dependence of the number density of created particles is a power law. Thereby in this model it is possible to produce superheavy relics with mass greatly exceeding the scalaron mass, which form supercold dark matter.

We start by recalling some relevant for the present study facts about  $R^2$ -inflationary model. The gravitational sector of this model is described by the following Lagrangian

$$S^{JF} = -\frac{M_P^2}{2} \int \sqrt{-g} d^4x \left( R - \frac{R^2}{6\mu^2} \right), \quad (1)$$

where  $R$  is scalar curvature,  $\mu$  is dimensionful parameter, and we introduce the reduced Planck mass  $M_P = M_{Pl}/\sqrt{8\pi} = 2.4 \times 10^{18}$  GeV. Variation of the action (1) with respect to

$g_{\mu\nu}$  yields the fourth order differential equation. One can argue (using general coordinate invariance) that the degrees of freedom of the field  $g_{\mu\nu}$  can be split into massless spin-2 field  $\tilde{g}_{\mu\nu}$  and massive scalar field  $\phi$  which obey the second order equations of motion [5]. This splitting is achieved explicitly in the action written in the Einstein frame, which one can come to by conformal transformation

$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \chi g_{\mu\nu} , \quad \chi = \exp \left( \sqrt{2/3} \phi / M_P \right) . \quad (2)$$

For the action (1) we have then

$$S^{EF} = \int \sqrt{-\tilde{g}} d^4x \left[ -\frac{M_P^2}{2} \tilde{R} + \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{3\mu^2 M_P^2}{4} \left( 1 - \frac{1}{\chi(\phi)} \right)^2 \right] , \quad (3)$$

everywhere  $\tilde{g}_{\mu\nu}$  stands for the metric tensor and  $\tilde{R}$  is scalar curvature for the metric  $\tilde{g}_{\mu\nu}$ . One observed that in the Einstein frame the action splits into the Einstein–Hilbert action for  $\tilde{g}_{\mu\nu}$  and ordinary action for the scalar field  $\phi$  with a specific potential. The scalar mode in the action (1) was named scalaron [1] and we will use this name for the field  $\phi$  in what follows.

The scalaron potential in (3) is exponentially flat at super-Planckian field values and provides the slow-roll inflation. Normalization of the amplitude of scalar perturbations, generated during this stage, to the observed CMB anisotropy requires [6]

$$\mu = 1.3 \times 10^{-5} M_P .$$

The spectral index and parameters of the generated tensor perturbations (for recent discussion see [7]) are consistent with observational constraints [8].

When the slow-roll conditions get violated, inflation terminates and the inflaton  $\phi$  starts to oscillate rapidly, with frequency equal to scalaron mass  $\mu$ . This drives the Universe expansion like at matter-dominated stage. The intermediate stage naturally ends up with inflaton decays into ordinary particles. Here we briefly discuss this process, see [1, 2, 3] for details.

The major role in the scalaron decay is played by scalars. Below we consider scalar  $\varphi$  described by the following Lagrangian in the Jordan frame

$$S_\varphi^{JF} = \int \sqrt{-g} d^4x \left( \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} m_\varphi^2 \varphi^2 + \frac{\xi}{2} R \varphi^2 \right) , \quad (4)$$

where the last term represents nonminimal coupling to gravity with dimensionless parameter  $|\xi| \lesssim 1$ . Note that with  $|\xi| \lesssim 1$  we do not introduce a new scale. Below we assume that  $\xi$  is

a free small parameter, but let us single out two special points: *minimal coupling* with  $\xi = 0$  and *conformal coupling* corresponding to  $\xi = 1/6$ . The first value leads to a free scalar field theory while the second one ensures conformal invariance of the action (4) without mass term. The mass term explicitly breaks conformal invariance and one can expect (small) deviation of  $\xi$  from the conformal value due to quantum corrections. Quantum corrections can also generate small but nonzero value of  $\xi$  for a scalar field minimally coupled to gravity. Therefore one can expect that the value of coupling  $\xi$  should be within some intervals covering  $\xi = 0$  or  $\xi = 1/6$  rather than be exactly equal to  $\xi = 0$  or  $\xi = 1/6$ , respectively. Below we are interested in only positive values of nonminimal coupling. Negative  $\xi$  may lead to a nonadiabatic evolution and thereby an amplification of scalar perturbations on superhorizon scales during inflation [9]. That in turn may change predictions for scalaron mass  $\mu$  and requires detailed study beyond the scope of this paper.

After conformal transformation (2) and field rescaling

$$\varphi \rightarrow \tilde{\varphi} = \chi^{-1/2} \varphi$$

the action (4) takes the form

$$S_{\varphi}^{EF} = \int \sqrt{-\tilde{g}} d^4x \left[ \frac{1}{2} \tilde{g}^{\mu\nu} \partial_{\mu} \tilde{\varphi} \partial_{\nu} \tilde{\varphi} + \frac{\xi}{2} \tilde{R} \tilde{\varphi}^2 - \frac{1}{2\chi} m_{\varphi}^2 \tilde{\varphi}^2 + \frac{1}{2} \left( \frac{1}{6} - \xi \right) \frac{\tilde{\varphi}^2}{M_P^2} \tilde{g}^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi + \sqrt{6} \left( \frac{1}{6} - \xi \right) \frac{\tilde{\varphi}}{M_P} \tilde{g}^{\mu\nu} \partial_{\mu} \tilde{\varphi} \partial_{\nu} \phi \right]. \quad (5)$$

One can find a conformal transformation involving  $\varphi$  and leading to another Einstein frame, where the nonminimal coupling disappears (see, e.g. [9]). This frame is more convenient to study the case of two-field inflation, where the dynamics of scalar field  $\varphi$  is also relevant at the inflationary epoch. However, below we are interested in a more simple situation, where only scalaron is responsible for the inflation, and adopt the metric (2).

Equation (5) describes the scalaron interaction with free scalars. It doesn't vanish at  $\xi = 0$  and provides scalaron decay into a pair of scalars. The second term in Eq. (5) yields on nontrivial cosmological background a contribution to the scalar mass. However, at reheating one has  $|\tilde{R}| \ll \mu^2$ , and the scalaron decay into scalars of mass  $m_{\varphi} < \mu/2$  remains kinematically allowed<sup>1</sup>. Its width reads [2]

$$\Gamma_{\xi} = \left( 1 - 6\xi + 2 \frac{m_{\varphi}^2}{\mu^2} \right)^2 \frac{\mu^3 \sqrt{1 - 4m_{\varphi}^2/\mu^2}}{192\pi M_P^2}. \quad (6)$$

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<sup>1</sup>At the inflationary stage the situation is different, as we discuss later on.

Massless scalar field with  $\xi = 1/6$  is conformally invariant and does not interact with the scalaron, which is illustrated by nullifying decay rate (6) at  $\xi \rightarrow 1/6$ ,  $m_\varphi \rightarrow 0$ . The mass term in Eq. (5) explicitly breaks conformal invariance and the scalaron decay rate in this case is proportional to  $m_\varphi^4$ , see Eq. (6). Hence, if the mass of the conformally coupled scalars is very small,  $m_\varphi \ll \mu$ , the scalaron decays very slowly.

The value of the coupling constant, which is natural and guarantees the successful reheating through scalaron decays, is  $\xi = 0$ . In case of the Standard Model with four scalar degrees of freedom (in the Higgs sector) minimally coupled to gravity scalaron rapidly decays to scalars thereby reheating the Universe upto the temperature [1, 2, 3]

$$T_{reh} \approx 3.1 \times 10^9 \text{ GeV} . \quad (7)$$

Now let us consider a new scalar field, which does not interact with SM particles or this interaction is very weak, so these particles never equilibrate in the primordial plasma. If stable at cosmological time scale, such scalar is a good candidate to be dark matter. Previously [3] we have shown that minimally coupled scalars are too hot to be dark matter. Here we rehabilitate light scalars as a possible dark matter candidate by introducing nonminimal interaction with gravity.

As we discussed above, light scalars are eventually produced from scalaron decays. In order to describe this process we will treat the scalaron field configuration as a condensate of nonrelativistic particles with number density  $n_\phi(t)$  and energy density  $\rho_\phi(t) = \mu n_\phi(t)$ . At some moment  $t_*$  the number density of scalar particles produced from scalaron decays is

$$dn(t_*) = 2 \Gamma_\xi n_\phi(t_*) dt_* = 2 \frac{\Gamma_\xi}{\mu} \rho_\phi(t_*) dt_* .$$

To get the number density at reheating time  $t_{reh}$  one should take into account the volume expansion,

$$dn(t_{reh}) = dn(t_*) \frac{a^3(t_*)}{a^3(t_{reh})} = 2 \frac{\Gamma_\xi}{\mu} \rho_\phi(t_*) \frac{a^3(t_*)}{a^3(t_{reh})} dt_* . \quad (8)$$

Now let us look at the physical momentum of these particles. At production time  $t_*$  it equals  $p_* = \sqrt{\mu^2/4 - m_\phi^2}$ . Then it gets redshifted and at reheating time one has

$$p_{reh} = p_* \frac{a(t_*)}{a(t_{reh})} . \quad (9)$$

Using this formula one can find the momentum difference at reheating time for the particles produced from  $t_*$  to  $t_* + dt_*$ ,

$$dp_{reh} = p_{reh} H(t_*) dt_* . \quad (10)$$

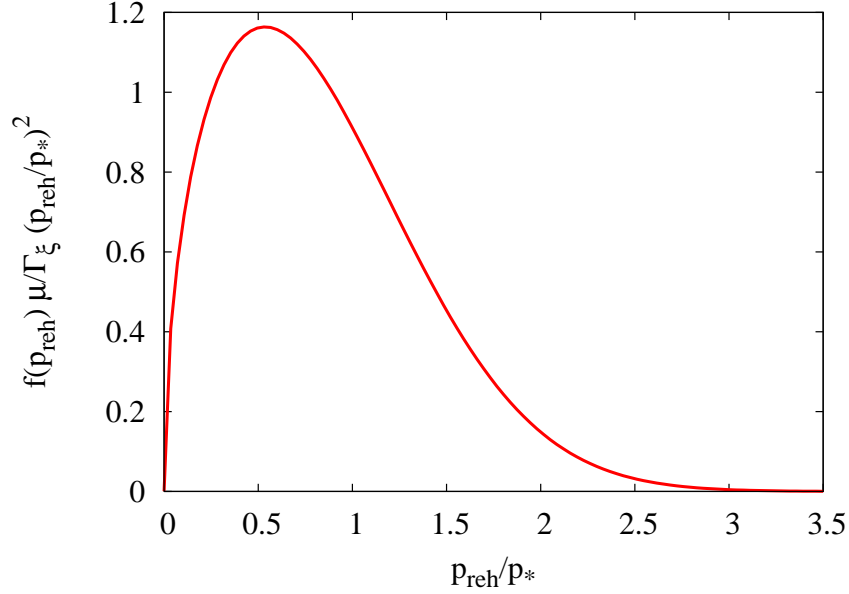


Figure 1: The distribution function for the scalars produced from scalaron decays.

On the other hand one can introduce the distribution function  $f(p_{reh})$  of produced particles as  $dn(t_{reh}) = f(p_{reh}) d^3p_{reh}/(2\pi)^3$ . Substituting Eqs. (8), (10) into this formula, we get

$$f(p_{reh}) = 4\pi^2 \frac{\Gamma_\xi}{\mu} \frac{\rho_\phi(t_*)}{p_{reh}^3 H(t_*)} \frac{a^3(t_*)}{a^3(t_{reh})}, \quad (11)$$

where production time  $t_*$  should be expressed in terms of momentum by Eq. (9). At  $t_* \ll t_{reh}$  one rewrites Eq. (11) as

$$f(p_{reh}) = 12\pi^2 \frac{\Gamma_\xi}{\mu} \frac{M_P^2 H(t_*)}{p_*^3}. \quad (12)$$

As one can see from Eq. (11), for calculation of the spectrum one needs to know the time dependencies of the scale factor, Hubble parameter and scalaron energy density. In order to obtain a more accurate expressions, we use numerical calculations. Our result for the spectrum of produced scalars is presented in Fig. 1.

For the particles number density one obtains then

$$n_\varphi \simeq 2.5 \frac{\Gamma_\xi}{\mu} M_P^2 H_{reh}, \quad (13)$$

where numerical factor in front of r.h.s. of Eq. (13) is the result of numerical integration of the spectra in Fig. 1 over momenta. For the average momentum one gets

$$\bar{p}_{reh} = 0.85 p_*. \quad (14)$$

Note that  $\bar{p}_{reh} \gg T_{reh}$ , so the particles are very hot in comparison with the plasma.

In order to find the present abundance of the dark matter one uses the ratio of the entropy to the number density. At the reheating time we have

$$\frac{s}{n(t_{reh})} \simeq 0.2 \frac{\pi \sqrt{g_*} \mu T_{reh}}{\Gamma_\xi M_P}, \quad (15)$$

where  $g_*$  is the effective number of degrees of freedom in the plasma, see e.g. [10]. The ratio (15) remains intact during the subsequent hot stages of the Universe expansion including the present epoch. From the requirement that the relative contribution of the dark matter to the present energy density  $\rho_c$  is [8]  $\Omega_{DM} = 0.223$  one gets the following relation between the particle mass and nonminimal coupling  $\xi$  which enters Eq. (6),

$$m_\varphi \simeq 0.2 \frac{\Omega_{DM} \rho_c}{s_0} \frac{\pi \sqrt{g_*} \mu T_{reh}}{\Gamma_\xi M_P}. \quad (16)$$

Finally let us find the lower and the upper bounds on the scalar mass. The lower bound corresponds to the case of warm dark matter. These particles should have the average velocity  $v_{eq}^{max} \sim 10^{-3}$  at the epoch of equilibrium between radiation and matter densities. On the other hand, the average velocity at that time can be found from Eq. (14). That leads to the particle mass

$$m_{\varphi, min} \simeq \frac{0.42}{v_{eq}^{max}} \left( \frac{g_{*,eq}}{g_*} \right)^{\frac{1}{3}} \frac{T_{eq}}{T_{reh}} \mu \simeq 1.1 \text{ MeV}, \quad (17)$$

where at the equilibrium  $T_{eq} \approx 0.76 \text{ eV}$ ,  $g_{*,eq} \approx 3.9$  and  $g_* = 106.75$  [10]. Then Eq. (16) implies  $\xi = 1/6 \pm 0.018$  for the viable dark matter candidate.

The upper bound can be imposed if we take into account the gravitational particle production from vacuum fluctuations in the expanding Universe. It has been shown [11, 12] that the particles conformally coupled to gravity and heavier than  $m_{\varphi, max} \simeq 10^9 \text{ GeV}$  overclose the Universe. Possible dark matter masses (16) together with the lower and upper bounds are shown in Fig 3.

The aforesaid consideration is valid while the occupation number for produced particles remains small,  $f(p_{reh}) \lesssim 1$ . Otherwise, coherent effects become important and the corresponding occupation number undergoes exponential growth. Let us check the maximum occupation number, which in our case is reached by the particles produced at the end of inflation as one can see from Eq. (12). Using the relation (16) one finds that the maximal occupation number exceeds unity only for the very light dark matter particles with

$$m_\varphi \lesssim 1 \text{ keV}. \quad (18)$$

These values of  $m_\varphi$  are smaller than (17), hence the coherent effects become significant for the irrelevant hot dark matter only, and our result (16) remains intact.

We proceed to the case, when dark matter particle is heavier than the half-scalaron. Perturbative production of such particles is kinematically forbidden. However, some amount of them is eventually produced in a time-dependent scalaron background. In order to describe this process one can use the method based on the Bogoliubov's transformation coefficients [13]. Here we summarize the basic formalism which we employed in our analysis. For more details see, e.g Ref. [14].

We start by canonically quantizing the action (5) in curved cosmological background with the external classical scalaron field. For this purpose it's convenient to choose the FLRW metric in the form  $ds^2 = a^2(\eta)(d\eta^2 - d\vec{x}^2)$ , where  $a(\eta)$  is the scale factor and  $\eta$  is the conformal time defined as  $d\eta = dt/a$ . After rescaling the field variable,

$$\tilde{\varphi} = s/a(\eta) ,$$

the equation of motion following from the action (5) becomes

$$\left\{ \frac{\partial^2}{\partial \eta^2} - \frac{\partial^2}{\partial \vec{x}^2} + \frac{1}{\chi} a^2 m_\varphi^2 - \left( \frac{1}{6} - \xi \right) \left( 6 \frac{a''}{a} + \frac{\phi'^2}{M_P^2} + \frac{\sqrt{6} a^2}{M_P} \frac{\partial V(\phi)}{\partial \phi} \right) \right\} s(\eta, \vec{x}) = 0 , \quad (19)$$

where prime means the derivative with respect to conformal time and  $V(\phi)$  is the scalaron potential, see Eq. (3). Here we have exploited the classical equation of motion to express the second conformal time derivative of the scalaron field. Solution of Eq. (19) can be written in the following form

$$s(\eta, \vec{x}) = \frac{1}{(2\pi)^{3/2}} \int d^3p \left( \hat{a}_p s_p(\eta) e^{-i\vec{p}\vec{x}} + \hat{a}_p^\dagger s_p^*(\eta) e^{i\vec{p}\vec{x}} \right) , \quad (20)$$

where  $\hat{a}_p$  and  $\hat{a}_p^\dagger$  are annihilation and creation operators, and the mode function  $s_p(\eta)$  obeys the oscillatory equation

$$s_p'' + \omega^2(\eta) s_p = 0 \quad (21)$$

with time-dependent frequency

$$\omega^2(\eta) = p^2 + \frac{1}{\chi} a^2 m_\varphi^2 - \left( \frac{1}{6} - \xi \right) \left( 6 \frac{a''}{a} + \frac{\phi'^2}{M_P^2} + \frac{\sqrt{6} a^2}{M_P} \frac{\partial V(\phi)}{\partial \phi} \right) . \quad (22)$$

While  $\omega$  varies adiabatically,  $|\omega'|/\omega^2 \ll 1$ , solution (20) corresponds to a particle-like field configuration (note that adiabatic field evolution is a necessary condition for introducing



the very idea of particles). At this stage the particle creation does not occur. The number density of dark matter particles is given by

$$n_\varphi = \frac{1}{(2\pi a)^3} \int d^3p |\beta_p|^2, \quad (23)$$

where  $\beta_p$  is the Bogoliubov's transformation coefficient, which relates initial and final particle-like states of the quantum field. It can be expressed in terms of the mode functions as

$$|\beta_p|^2 = \frac{|s'_p|^2 + \omega^2 |s_p|^2}{2\omega} - \frac{1}{2}. \quad (24)$$

Assuming initially no dark matter in the Universe we impose the vacuum initial conditions

$$s_p \rightarrow 1/\sqrt{2\omega}, \quad s'_p \rightarrow -i\omega s_p, \quad \text{at } \eta \rightarrow -\infty. \quad (25)$$

As mentioned above, the particle number is conserved during adiabatic evolution. On the contrary, highly varying frequency gives rise to particle production and the higher the variation rate the larger the number of produced particles. In our case the maximal violation of the adiabaticity occurs right at the end of inflation due to the scalaron-dependent mass term. Particle creation in the case of large  $m_\varphi$  can be especially sensitive to scalaron dynamics at this stage. Therefore we resort to numerical methods in order to find exact evolutions of the scalaron field and the scale factor, and further solve numerically Eqs. (21), (22) for the mode function. Because of the limited capability of numerical calculations we impose the initial conditions (25) not at the past infinity but at some finite value of  $\eta$ . We found that the results become practically independent of the choice of the initial time corresponding to more than 7–8 e-foldings till the end of inflation. It was also learned from numerical simulations that the number of particles in comoving volume produced at the end of inflation remains almost constant during subsequent evolution. Indeed, the number of produced particles is very small, hence no any Bose-enhancement is expected during the subsequent evolution. On the other hand, both inflaton velocity and the Hubble parameter slow down as the Universe evolves at the post inflationary matter-dominated stage. As one can see from (22), this leads to strengthening of adiabaticity and very low rate of particle production after the end of inflation.

A convenient quantity for subsequent computation is the ratio of the energy density of produced scalar particles to the total energy density. This ratio remains almost constant while the Universe expands due to oscillating scalaron. Numerical results for this ratio in the case of minimally coupled scalars,  $\xi = 0$ , are presented in Fig. 2 (small circles). It turns

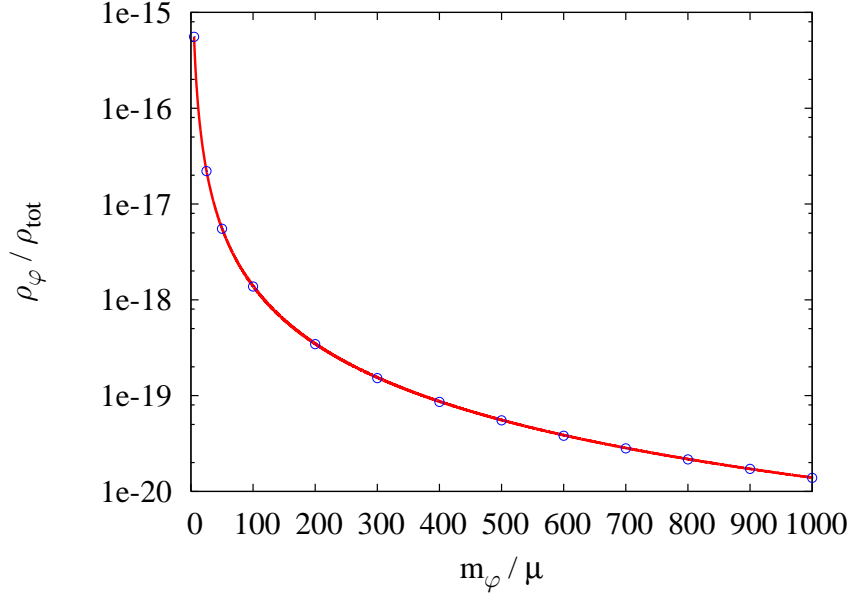


Figure 2: The ratio of the energy density of produced scalar particles minimally coupled to gravity to the total energy density. Small circles represent numerical results, while the line corresponds to numerical fit with the formula (26).

out that numerical data are well described by the relation

$$\frac{\rho_\varphi}{\rho_{\text{tot}}} = \alpha(\xi) \left( \frac{\mu}{m_\varphi} \right)^2, \quad (26)$$

which is plotted in Fig. 2. Here the value of  $\alpha$  for a given  $\xi$  we obtained by fitting numerical data with the formula (26). We found that  $\alpha$  varies from  $10^{-14}$  to  $10^{-16}$ , when  $\xi$  varies from 0 to  $1/4$ . Note, that the resulting number density of produced particles exhibits power law mass-dependence. This differs from the cases of superheavy particle production by pure gravity (see, e.g. [11, 12]) or by combined effects of gravity and inflaton field with polynomial coupling to the scalars (see, e.g. [15]), where the mass dependence of particles number density is exponential as mass exceeds expansion rate. In our case at inflationary stage the effective mass of scalars determined by the inflaton field is very small: i.e. they are not superheavy. Number density of produced at this stage scalars are exponentially decreases due to inflation. Scalars become superheavy at  $\phi \lesssim M_P$  when inflation terminates. The scalar mass depends exponentially on the value of scalaron field (see Eq. (5)). Consequently the particle production is saturated at the end of inflation, when scalaron-dependent mass rapidly changes from  $m_\varphi \ll H$  to  $m_\varphi > H$  violating adiabaticity, and exponential decrease

in the number density due to the Universe expansion does not occur. From this chain of reasonings we conclude that it is exponentially-changing mass term which is responsible for the power law mass dependence (26). It would be worth, however, to find more rigorous arguments (analytic calculations) to support the numerical result (26).

Making use of Eq. (26) and the requirement that produced scalar particles should explain dark matter phenomenon, one gets the relation between the dark matter mass and the nonminimal coupling constant,

$$m_\varphi = \mu \left( \frac{3}{2} \alpha(\xi) \frac{s_0 T_{reh}}{\Omega_{DM} \rho_c} \right)^{1/2}. \quad (27)$$

The lighter particles are overproduced in this model in accordance with Eqs. (26), (27). However, for very light scalars,  $m_\varphi < \mu/2$ , Eq. (26) is not applicable. We find numerically in this case that the scalar production rate actually goes down with decreasing  $m_\varphi$  so that overproduction of particles with mass of two or three orders of magnitude lighter than the scalaron does not occur. However in this region of masses the dominant process is the perturbative production at reheating stage given by Eq. (6).

Our final results for possible masses of scalar dark matter given by Eqs. (16), (27) are shown in Fig. 3 by solid thick lines. The top of the figure corresponding to superheavy dark matter particles with  $m_\varphi \geq \mu/2$  is plotted in linear scale. Here the  $\xi$ -dependence of dark matter mass is almost linear as one can see from the figure. The filled area above the curve represents cosmologically allowed masses for any free scalars in this model, which do not overclose the Universe.

The case of light dark matter which masses are given by Eq. (16) is presented in logarithmic scale. The zoom shows the fine structure of the curve near conformal point  $\xi = 1/6$ . The filled region under the curve represents allowed masses of free scalar particles in the model. Allowed region is bounded from above by dashed line corresponding to scalar mass of  $10^9$  GeV. This bound appears when we consider all possible mechanisms of particle production which are relevant for this masses. Indeed, from Fig. 3 one concludes that the cosmologically allowed free scalars with  $m_\varphi \lesssim 10^{12}$  GeV are only those which are almost conformal. On the other hand, the scalar particles conformally coupled to gravity are produced in expanding Universe from vacuum fluctuations and overclose the Universe if  $m_\varphi \gtrsim 10^9$  GeV [12]. The mass range  $10^{12}$  GeV  $\lesssim m_\varphi \lesssim \mu/2$  is excluded, if we take into account particle production during inflation, see the discussion below Eq. (27). Note, that limits obtained in [12] are not applicable to the model with nonminimal coupling outside the conformal window  $\xi \approx 1/6$  and scalar mass approaching  $\mu/2$  from below, see Fig. 3, which we do not study

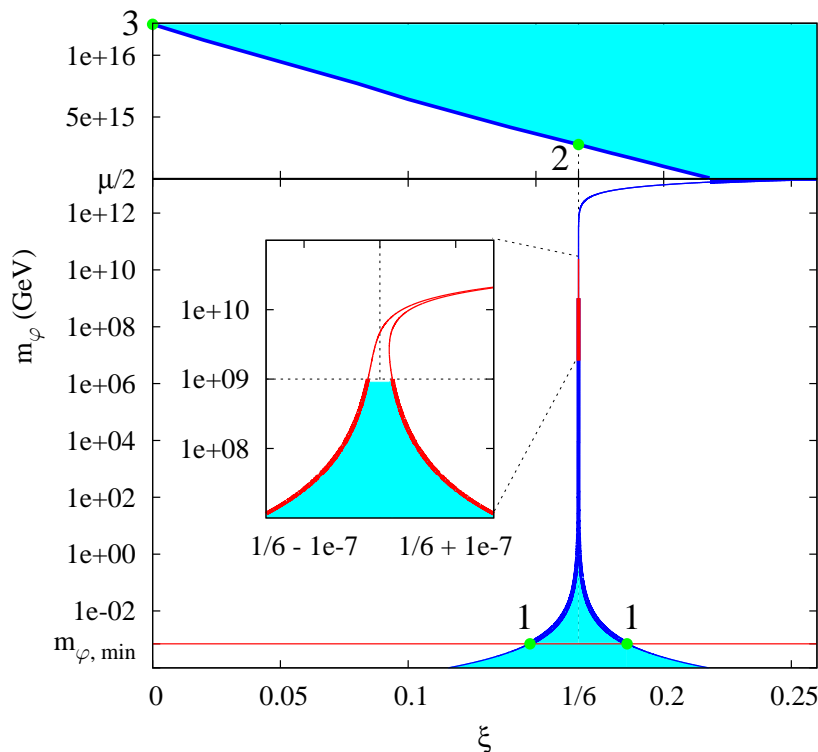


Figure 3: Mass of the scalar dark matter candidates as a function of nonminimal coupling constant  $\xi$  (solid thick lines). The filled area represents cosmologically allowed masses of free scalars. At the bottom part of the plot the allowed region is located between horizontal lines corresponded to  $m_\varphi \simeq 10^9$  GeV and  $m_\varphi \simeq 1.1$  MeV. The zoom shows the fine structure of the curve near the conformal point  $\xi = 1/6$ .

in this work. Also, light free scalars of  $m_\varphi \ll 1$  keV may be forbidden for  $\xi$  in particular regions, where the exponential amplification of scalar production is expected due to coherent effects, see Eq. (18) and discussion therein.

The special choices of dark matter parameters are marked by numbers in Fig. 3. The points with label “1” correspond to warm dark matter with mass given by Eq. (17). The point “2” represents conformally coupled scalars with  $m_\varphi \simeq 2.8 \times 10^{15}$  GeV. The free dark matter scalars have  $m_\varphi \simeq 1.3 \times 10^{16}$  GeV, which is marked by “3” in Fig. 3.

Note in passing, that free superheavy fermions are also produced during inflation. We calculate their number density as we did for scalars. In order to be dark matter these fermions should have mass  $m_\psi \simeq 3.1 \times 10^{15}$  GeV, which is similar to the case of conformally coupled scalars presented above. It is worth to emphasize, that superheavy relics produced

at the end of inflation, if unstable, cannot help to reheat the Universe earlier. As we have seen from Fig. 2, these particles take away a very small fraction of the energy. One can also evaluate their number density to entropy ratio. For example, for the fermions of  $10^{12}$  GeV at reheating we have  $n_\psi/s \sim \alpha \sim 10^{-15}$ . This value is much smaller than  $\eta_B$ . Therefore, seesaw sterile neutrinos produced by this mechanism cannot help with leptogenesis and one must exploit the perturbative production of seesaw sterile neutrinos by the scalaron decay [3], to explain the neutrino oscillations and the baryon asymmetry of the Universe.

To conclude, we have shown that in  $R^2$ -inflation the scalar particles can form dark matter, which may be warm, cold or supercold depending on scalar mass and the value of nonminimal coupling constant to gravity (see Fig. 3).

We end up with several comments. First of all, note that introduction of selfinteracting terms for the scalars can change our predictions for dark matter masses. Indeed, for the scattering processes which preserve the particles number, the average momentum is conserved and our predictions remain unchanged. However, if selfinteraction is strong enough so that scattering to multi-particle final states becomes rapid, the particle number density increases while average momentum goes down. In order to keep constant the relative contribution of the dark matter to the present energy density, one should decrease the mass of the candidate for a given value of  $\xi$ . This decreasing factor for the mass is equal to the decreasing factor for the average momentum so that the average velocity at the epoch of equilibrium between radiation and matter densities would be the same. This in particular means that the boundary values of nonminimal coupling constant (marked by “1” in Fig. 3) will refer to the warm dark matter case as before.

Second, superheavy dark matter, if slowly decaying, can show up in ultra-high energy cosmic rays. The cosmologically long but finite lifetime can be explained by a nonperturbative mechanisms of their decays, such as in the instanton scenarios [16], or involving quantum gravity (wormhole) effects [17].

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